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Iterative learning control for Linear Discrete Time Switched Systems

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Abstract. This paper considers the stability analysis and the stabilization problem for a class of discrete-time switched Systems. A sufficient condition based on the average dwell time that guarantees the exponential stability of switched linear systems is given. First, the iterative learning control (ILC) is presented to build a formulation ensuring the exponential stability of the given system. The results were obtained through original connection with the notion of stability along the pass for 2 D repetitive system. All the results are presented in terms of linear matrix inequalities (LMIs). A numerical simulation example is established shown the effectiveness of the proposed method.

Keywords: Switched systems; Iterative learning control (ILC); 2D systems; stability Roesser model ; Dwell time ;Linear matrix inequality.

1 Introduction

The stability analysis of switched linear systems is one of the active over the past research due to their wide applications in many areas such (the automotive industry, aircraft and air-traffic control, switched power converters) [1, 2]. A switched system is a hybrid dynamical system, which consists of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching 1between them [3, 4, and 5]. For the stability analysis of switched systems there are two issues to be addressed: stability under arbitrary switching [6, 7, 8, and 9] and stability under constraint switching, such as dwell time, average dwell time constraints (i.e. minimum time between switching) [10, 11]. However, the stabilization problem of discrete switched systems with classic control law has been studied in [8, 9, 12, 13, 14,15], by using the common Lyapunov function approach, multiple Lyapunov function approach, piecewise Lyapunov function and switched Lyapunov function [16,17,18]. Many results on the issues of stability and control synthesis for the 2D switched systems have been obtained in [19, 20], most of them are based on the dwell time approach. Specifying the minimum dwell time that ensures the exponential stability of the switched systems is an especially attracting problem in this paper. Compared with the different results for synthesis issue by applying the classic control laws, relatively few efforts are made for designing a controller to achieve the exponential stability for switched repetitive systems. This paper is addressed for the design feedback controllers (ILC) such that the system is stable [21, 22, 23, 24, and 25]. Motivated by human learning, the basic idea of (ILC) is to use information from previous executions of the task in order to improve performance from pass-to-pass in the sense that the tracking error is sequentially reduced. Iterative learning control (ILC) is an attractive technique when it comes to dealing with systems that execute the same task repeatedly over a finite time interval. This method of control uses information from previous executions of the task in an attempt to improve performance from repetition to repetition in the sense that the system is exponentially stable. The objective of ILC schemes is to use their repetitive process structure (i.e. information propagation from trial-to-trial (or pass-to-pass) and along a trial/pass) to progressively improve the accuracy with which the core operation under consideration is performed, by updating the control input progressively from trial-to-trial.

However, to the best of our knowledge, the problems of stability and stabilization for switched systems via the dwell time approach shows good results in practice, especially for the exponential stability problem with (ILC) control of switched systems, which motivates our present study. In this paper, the problem of iterative learning control for a class of linear discrete- time switched system is studied. The obtained formulation by applying ILC control is transformed into a synthesis problem of a special 2D Roesser switched system [13, 26]. In order, to describe and to analyze the performance of the system along the time and along the cycle, new concepts and convergence indices are introduced to describe the dynamical behaviors of the corresponding discrete switched system in horizontal and vertical propagation directions. Sufficient conditions for stability of the 2D Roesser system are derived [9]. One is faced with determining the minimum dwell time such that exponential switching is maintained [10]. It is well known that the computation of the exact minimum dwell time is a problem which must be overcome. This paper is organized as follows: In section 2, based on the stability and stabilization of 2D Roesser discrete switched systems mathematical settings have been addressed. In section 3, the control law design (ILC) by constructing a sequence of control inputs to a discrete switched system produces 2D repetitive system, and sufficient conditions for the existence of a stabilizing controller (ILC) are derived in terms of a set of matrix inequalities. In section 4, Numerical examples are presented to illustrate the feasibility and the effectiveness of the proposed design algorithms in this paper. Finally, the conclusion of this paper is given in Section 5.

Notations- Throughout this paper, the superscript "T" denotes the transpose, the notation $(X \geq 0, X > 0)$ is positive semi-definite (positive definite, respectively). Matrix $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P , respectively. I represents identity matrix with appropriate dimension. The asterisk $*$ in a matrix is used to denote a term induced by symmetry. The set of all nonnegative integers is represented by Z_+ .

2 Exponential stability of 2D repetitive switched system

Consider the following 2D-discrete linear switched system described by the Roesser model:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A_{\sigma(i,j)} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bu(i,j) \quad (1)$$

$$\begin{cases} x^h(i,j) = \begin{cases} f(j), & 0 \leq j < r_1 \\ 0 & j > r_1 \end{cases} \\ x^v(i,0) = \begin{cases} g(i), & 0 \leq i < r_2 \\ 0 & i > r_2 \end{cases} \end{cases}, \{i,j\} \in k =: \{0,1...,n\} \quad (2)$$

Where $x^h(i,j) \in IR^{n_1}$ is the horizontal state in IR^{n_1}, $x^v(i,j) \in IR^{n_2}$ is the vertical state in IR^{n_2}, $x(i,j) \in IR^n$ with $n = n_1 + n_2$ is the whole state in IR^n, $u(i,j) \in IR^m$ is the control vector in IR^m, $\sigma(i,j)$ is a switching signal which takes its values in the finite set $\boldsymbol{N} = \{0,..,N\}$, $i, j \in Z_+$, N is the number of subsystems.

$$A_{\sigma(i,j)} = \begin{bmatrix} A_{11}^{\sigma(i,j)} & A_{12}^{\sigma(i,j)} \\ A_{21}^{\sigma(i,j)} & A_{22}^{\sigma(i,j)} \end{bmatrix}$$

$$A_{11}^{\sigma(i,j)} \in IR^{n_1 \times n_1}, A_{12}^{\sigma(i,j)} \in IR^{n_1 \times n_2}, A_{21}^{\sigma(i,j)} \in IR^{n_2 \times n_1}, A_{22}^{\sigma(i,j)} \in IR^{n_2 \times n_2}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, B_1 \in IR^{n_1 \times m}, B_2 \in IR^{n_2 \times m}, k \in N$$

Are known constant matrices with appropriate dimensions.

In the sequel, sufficient condition of exponential stability of 2D discrete switched system described by Roesser structure (1) and (2) is given.

***Definition 1*:** [9] System (1) is said to be exponentially stable under $\sigma(i,j)$ if there exists a positive constant c for a given $r \ge 0$, such that:

$$\sum_{i+j=D} \left\| x(i,j) \right\|^2 < \xi e^{-c(D-r)} \sum_{i+j=r} \left\| x(i,j) \right\|^2, D \ge r, \xi \ge 0 \tag{3}$$

Holds for all $D \ge r$ and a positive constant ξ.

Remark1: From the definition 1, it is easy to see that when r is given, $\sum_{i+j=r} \left\| x(i+j) \right\|^2$ will be bounded, and $\sum_{i+j=D} \left\| x(i,j) \right\|^2$ will tend to zero exponentially as D goes to infinity, it means that $\left\| x(i,j) \right\|^2$ will tend to zero.

***Definition 2*:** [10] for any $i+1 = D \ge r = i_r + j_r$, let $N_\sigma(r,D)$ denote the switching number of $\sigma(i,j)$ on an interval (r,D),if:

$$N_\sigma(r,D) \le N_0 + \frac{D-r}{\tau_a} \tag{4}$$

With $N_0 \ge 0, \tau_a \ge 0$, τ_a is called the average dwell time, N_0 is the chatter bound.

Now, we present the problem of exponential stability analysis for the 2D discrete linear switched systems described by the Roesser model .

Theorem1[10] Consider system (1) and (2) with $u(i,j) = 0$,for a given positive constant $\alpha < 1$,if there exist positive definite symmetric matrices X_1^k, X_2^k with appropriate dimensions, $k \in N$ such that.

$$\begin{bmatrix} -\alpha X_1^k & 0 & \left(A_{11}^k X_1^k\right)^T & \left(A_{21}^k X_1^k\right)^T \\ * & -\alpha X_2^k & \left(A_{12}^k X_2^k\right)^T & \left(A_{22}^k X_2^k\right)^T \\ * & * & -\left(X_1^k\right)^T & 0 \\ * & * & * & -\left(X_2^k\right)^T \end{bmatrix} < 0 \tag{5}$$

Then, under the following average dwell time scheme:

$$\tau_a \ge \tau_a^* = \frac{\ln X}{-\ln \alpha} \tag{6}$$

$$X = \max_{\substack{1\le k,l\le N \\ k\ne l}} \frac{\lambda_{\max}\left(X^l\right)}{\lambda_{\min}\left(X^k\right)},\ X^k = diag\left(X_1^k, X_2^k\right) \tag{7}$$

In the next section, we propose a discrete iterative learning control algorithm (ILC) ensuring the exponential stability of discrete switched systems described by 2D Roesser models.

3 Exponential stabilization of linear switched system using ILC design

Consider the following discrete linear switched system described by the following form:

$$x(t+1) = A_{\sigma(t)}x(t) + Bu(t), x(0) = 0, t \ge 0 \tag{8}$$

Where $x(t) \in IR^n$ is the state in IR^n , $u(t) \in IR^m$ is the control vector in IR^m , $\sigma(t)$ is a switching signal which takes its values in the finite set $\boldsymbol{N} = \{0,..,N\}$, N is the number of subsystems. With $A_{\sigma(t)}$ is constrained to jump among the N vertices of the matrix polytope $\{A_1.......A_N\}$, B is matrix of appropriate dimension.

In the sequel, if k is defined as the switching number, Thus (8) can be presented in the following 2-D switched form:

$$x_k(t+1) = A_{\sigma(t)}x_k(t) + Bu_k(t), x(0) = 0, t \ge 0 \tag{9}$$

Where, $x_k(t)$ and $u_k(t)$ are respectively the state and the control vector of system in switching k . $t = 0,....,T$, $k \in IN$.

The stabilizability of the switched discrete time system (8) is investigated by the (ILC) approach, the latter is based on the computing of the difference between control law u at the respectively iteration k and $k+1$ .The system is required to execute the repetitive task over the finite time interval $0 < t < T$. In the kth operation/iteration, the control input, $u_k(t)$ may be updated iteratively in a certain way by using the state measurements in the previous operation. The switched system (1) admits a family of solutions that is parameterized both by the initial condition $x_k(0)$ and the switching signal. The proposed ILC algorithm can be

viewed as an extension form developed for 2-D switched systems, by considering the average dwell time between switching instants repeated at each iteration.
According to the principle of ILC, there are two independent dynamic processes: system time t, and learning iteration k during learning process. Every variable can be expressed as a 2-D switched function, such as $x_k(t)$, which represents $x(t)$ in the kth learning iteration (kth switching).
Consider the general ILC law given by:

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t) \tag{10}$$

Where, $\Delta u_{k+1}(t)$ denotes the variation of the control input at the respectively iteration k and $k+1$.
Firstly, an updating law based on the ILC approach described by the following form is used:

$$u_{k+1}(t) - \beta u_k(t) = K_1^p x_k(t) + K_2^p \eta_{k+1}(t) \tag{11}$$

With p is defined as the number of subsystems, if k is defined as the switching number.
Where

$$\eta_{k+1}(t) = x_{k+1}(t) - \beta x_k(t) \tag{12}$$

K_1^p, K_2^p are appropriately dimensioned matrices gains to be designed, and β a positive scalar.

Then clearly (11) and (12) can be written as:

$$\begin{aligned} \eta_{k+1}(t+1) &= x_{k+1}(t+1) - \beta x_k(t+1) \\ &= A_{k+1}x_{k+1}(t) + Bu_{k+1}(t) - \beta\left(A_k x_k(t) + Bu_k(t)\right) \\ &= A_{k+1}\eta_{k+1}(t) + \left(\beta\left(A_{k+1} - A_k\right) + BK_1^p\right)x_k(t) + \left(BK_2^p \eta_{k+1}(t)\right) \\ &= \left(A_{k+1} + BK_2^p\right)\eta_{k+1}(t) + \left(\beta\left(A_{k+1} - A_k\right) + BK_1^p\right)x_k(t) \end{aligned} \tag{13}$$

And

$$x_{k+1}(t) = \eta_{k+1}(t) + \beta x_k(t) \tag{14}$$

The obtained system in the closed-loop is represented by the following form:

$$\begin{cases} \eta_{k+1}(t+1) = \left(A_{k+1} + BK_2^p\right)\eta_{k+1}(t) + \left(\beta\left(A_{k+1} - A_k\right) + BK_1^p\right)x_k(t) \\ x_{k+1}(t) = \eta_{k+1}(t) + \beta x_k(t) \end{cases} \tag{15}$$

Let $A_{11} = \left(A_{k+1} + BK_2^p\right), A_{12} = \left(\beta\left(A_{k+1} - A_k\right) + BK_1^p\right), A_{21} = I, A_{22} = \beta$

However, the switched system (15) obtained by applying ILC control can be rewritten into 2D discrete linear switched system form described by the Roesser model (1) with:

$$\begin{cases} x^h(i+1,j) = \eta_k(t+1) \\ x^v(i,j+1) = x_{k+1}(t) \end{cases} \tag{16}$$

Via the Lyapunov characterization of stability along the trial, LMI algorithms are available to guarantee the exponential stability of linear discrete switched systems. The ILC scheme applied to the 2D Roesser switched systems required the feasibility of the following set of LMIs.

***Theorem 2*:** Discrete linear switched system (8) under (ILC) control law (11) is exponentially stabilizable, if there exist symmetric positive matrix $X_1^p, X_2^p, R_1^p, R_2^p, p \in Z_+$ and $0<\alpha<1$, $0<\beta<1$ such that the following LMI is feasible:

$$\begin{bmatrix} -\alpha X_1^p & 0 & \left(X_1^p\right)^T (A_{k+1})^T + \left(R_1^p\right)^T (B)^T & \left(X_1^p\right)^T \\ * & -\alpha X_2^p & \left(X_2^p\right)^T (\beta(A_{k+1}-A_k))^T + \left(R_2^p\right)^T (B)^T & \left(\beta X_2^p\right)^T \\ * & * & -\left(X_1^p\right)^T & 0 \\ * & * & * & -\left(X_2^p\right)^T \end{bmatrix} < 0 \tag{17}$$

In this case K_1^p and K_2^p are given by $K_1^p = R_2^p \left(X_2^p\right)^{-1}, K_2^p = R_1^p \left(X_1^p\right)^{-1}$ and the minimum dwell time given by:

$$\tau_a \geq \tau_a{}^* = \frac{\ln X}{-\ln \alpha}\text{ , With } X = \max_{\substack{1\leq k,l\leq N \\ k\neq l}} \frac{\lambda_{\max}\left(X^l\right)}{\lambda_{\min}\left(X^p\right)},\; X^p = diag\left(X_1^p, X_2^p\right)$$

Proof: Appling theorem1 with,

$$A_{11} = \left(A_{k+1} + BK_2^p\right), A_{12} = \left(\beta\left(A_{k+1} - A_k\right) + BK_1^p\right), A_{21} = I, A_{22} = \beta.$$

And substitute it into (5); we obtained the following LMI condition (17).

The stabilizability problem using the iterative learning control for a class of linear discreet switched system is not study until now. The application of the iterative learning control (ILC) which is a technique for design control systems operating in a repetitive (or pass-to-pass) mode allows to ensure the exponential stability of the closed-loop switched system and the convergence along the time and the pass. Sufficient conditions of exponential stability for the considered discrete switched system are also proposed in (17). It is worth noting that the given conditions in Theorem 2 is obtained by using the average dwell time approach and the parameters α and β play a key role in obtaining the minimum average dwell time τ_a .

4 Numerical examples

To prove the effectiveness of the proposed method, a numerical evaluation is discussed in this section. The studied problem consists in the design of an Iterative learning controller (ILC) stabilizing the discrete switched system. It is assumed that the switched system is operated during a finite time interval repetitively, and then the ILC control scheme can be used to achieve the convergence of the switched system over the whole time interval. A sufficient condition of exponential stability is announced for the considered switched system with ILC control in (11)–(12) minimizing the dwell time needed to overcome the stability analysis problem. Theoretical results are demonstrated by numerical simulations.

Consider the discrete-time switched system (1) governed by the two instable modes described by the following set of matrices:

$$A_1 = \begin{bmatrix} 5 & 1.5 \\ 1.4 & 2.6 \end{bmatrix}, A_2 = \begin{bmatrix} 8.5 & 5 \\ 3 & 2.5 \end{bmatrix}, B = \begin{bmatrix} 9 \\ 3.5 \end{bmatrix}$$

Applying the proposed design ILC scheme to the given system required the solving of the following LMIS (17) in order to obtain the stabilizing gains and the minimum dwell time assuring the exponential stability of the closed loop repetitive switched system.

The obtained results give an idea about the estimation of the minimum dwell time in order to validate the theoretical approach:

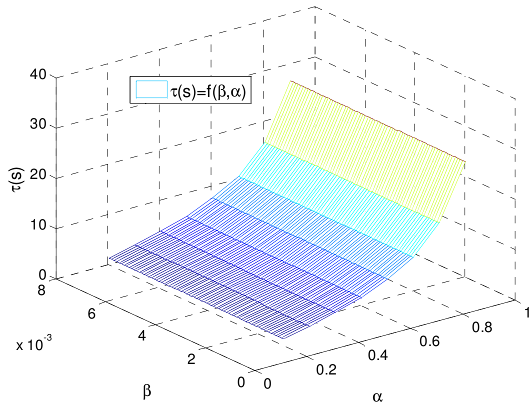

Fig. 1. Value optimization α, β according to minimum dwell time.

This figure represents the evolution of dwell time as a function of the constant β and the convergence rate α .

We choose $\alpha = 0.11, \beta = 0.007$, solving the matrix inequalities in Theorem 2 give rise to:

$K_1^1 = [0.004, -0.005]; K_1^2 = [-0.0028, -0.0025]; K_2^1 = [-0.1393, -1.7132]\ ; K_2^2 = [-0.9365, -0.5699]$
With $\tau_a = 4.83\,s$.

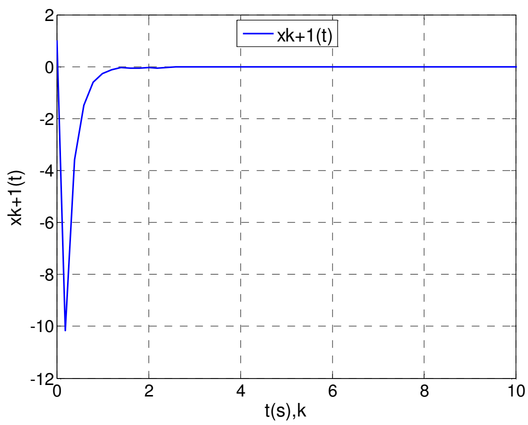

Fig. 2. The trajectory of state $x_{k+1}(t)$.

Now we represent the evolution of the command ILC along the time and cycle.

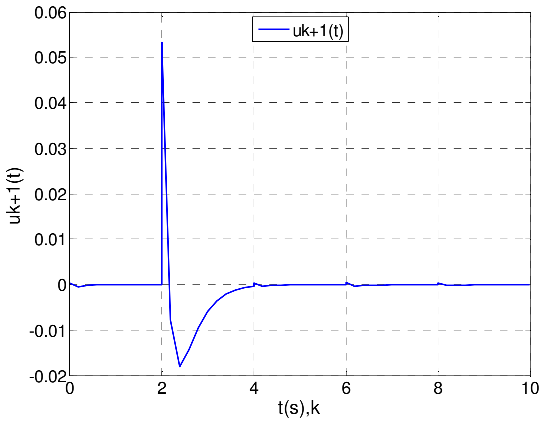

Fig. 3. The trajectory of the command $u_{k+1}(t)$.

Remark2 The simulation results shown in Figure.2, 3, by increasing the iteration number, the switched system is stable along the time and the pass, following selection of K_1^1, K_1^2, K_2^1 and K_2^2. Thus, the resulting ILC process guaranteed the minimum dwell time assuring the exponential stability of the closed loop repetitive switched system. The previous results show the effectiveness of the repetitive control ILC compared to other conventional control. The (ILC) control achieves the exponential stability of the studied closed-loop system.

5 Conclusion

In this paper, the problem of synthesis of (ILC) controller for a class of discrete-time linear switched systems under time switching laws has been investigated. The control law design (ILC) by constructing a sequence of control inputs to a discrete switched system produces 2D switched repetitive system. This can guarantee the exponential stability of the closed loop switched system and the convergence along the time and the pass. The dwell time approach is used for the stability analysis. Sufficient conditions for the existence of such controller are formulated in terms of a set of LMI. A numerical example is given to illustrate the effectiveness of the proposed method.

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